Summer Review for Students enrolling in AP Calculus BC.

As a student in Calculus, it is important that you have a thorough knowledge of skills in Algebra and PreCalculus. This review is optional and is only intended to help you review concepts necessary to be successful in BC Calculus. However, we do quiz on some of these concepts during the school year. This review is designed to be done without a calculator. We will give a graded review on these topics the first week of school.

1. Simplify: \( \frac{5(x + h)^3 - 5x^3}{h} \)  
2. Simplify: \( \frac{1}{x} + \frac{4}{x^2} \) 

3. Convert \( \frac{3a}{b} + b \) to a single fraction.

Factor each expression.

4. \( x^3 - 1 \)  
5. \( x^2 - 2 \)  
6. \( 6x^3 + 2x^2 - 4x \)

Solve for \( x \).

7. \( 2x^2 + 3x - 3 = 0 \)  
8. \( x^3 + 2x^2 - x - 2 = 0 \)

9. \( \sqrt{x^2 + 5} + x = 5 \)  
10. \( \frac{3x + 5}{(x - 1)(x^4 + 7)} = 0 \)

11. \( 6x^2 - 31x + 18 < 0 \)  
12. \( 2x^3 - 3x^2 - 2x + 3 > 0 \)

13. \( |x + 2| \leq 1 \)  
14. \( |x| > 10 \)

15. Find the equation of the line that passes through the points (1, -3) and (3, 2).

Give the domain of each of the following:

16. \( f(x) = \frac{1}{x - 2} \)  
17. \( f(x) = \sqrt{x^2 - x - 2} \)  
18. \( f(x) = \sqrt[3]{x - 5} \)

19. Find the x- and y-intercepts for \( f(x) = \frac{x^2 - 5}{2x + 3} \)
20. Sketch the graph of the piecewise function: \( f(x) = \begin{cases} -x & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x < 3 \\ -4 & \text{if } x > 3 \end{cases} \)

21. Let \( f(x) = 3x^2 + 1 \) and \( g(x) = \sqrt{x} \)

Find: A) \( f(g(x)) \) B) \( g(f(x)) \) and state the domain of each.

22. For \( f(x) = 2x - 1 \), find:

A) \( f(3) \) B) \( f(x + h) \) C) \( \frac{f(x + h) - f(x)}{h} \)

Evaluate:

23. \( \cos\left(\frac{\pi}{3}\right) \) 24. \( \csc\left(\frac{\pi}{4}\right) \) 25. \( \tan\left(\frac{\pi}{6}\right) \)

26. \( \sin\left(-\frac{\pi}{3}\right) \) 27. \( \cos\left(-\frac{\pi}{6}\right) \) 28. \( \arcsin\left(\frac{\sqrt{3}}{2}\right) \)

29. \( \arccos(-1) \) 30. \( \sec(\pi) \)

31. Find all values of \( \theta \) (where \( 0 \leq \theta < \frac{\pi}{2} \)) in terms of \( \pi \) for which \( \cos(6\theta) = 0 \).

32. Give the domain and range of \( y = \sin x \) and \( y = \cos x \).

33. Solve for \( x \): \( 2\sin^2 x - \sin x = 1 \) \( (0 \leq \theta < 2\pi) \)
Find the following limits.

34. \( \lim_{x \to 3} \frac{9x^2 - 1}{3x - 1} \)
35. \( \lim_{x \to 0} \frac{|x|}{x} \)
36. \( \lim_{x \to 0} \frac{|x|}{x} \)
37. \( \lim_{x \to 1} \frac{x - 1}{x^2 - 1} \)
38. \( \lim_{x \to 0} \frac{|x|}{x} \)
39. \( \lim_{x \to \infty} \frac{1 - \cos x}{x} \)
40. \( \lim_{x \to 0} \left( x^2 - \frac{1}{x} \right) \)
41. \( \lim_{x \to \infty} \frac{4}{x} \)
42. \( \lim_{x \to 3} f(x) \) when \( f(x) = \begin{cases} x^2 - 4x + 4, & x < 3 \\ -x^2 + 4x - 2, & x > 3 \end{cases} \)
43. Find the points where \( f(x) \) is discontinuous: \( f(x) = \frac{x^2 - x - 2}{x^2 - 1} \)
44. Find the equation of the vertical and horizontal asymptote(s), if any, for:
   \( f(x) = \frac{x^2 - 2x - 8}{x^2 - 4} \)
45. Find \( \lim_{h \to 0} \frac{\sin(x + h) - \sin(x)}{h} \)
Trigonometry things you should know from PreCalculus

Reciprocal Identities:

\[
\sin x = \frac{1}{\csc x} \quad \csc x = \frac{1}{\sin x} \\
\cos x = \frac{1}{\sec x} \quad \sec x = \frac{1}{\cos x} \\
\tan x = \frac{1}{\cot x} \quad \cot x = \frac{1}{\tan x}
\]

Quotient Identities:

\[
\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}
\]

Pythagorean Identities:

\[
\sin^2 x + \cos^2 x = 1 \\
1 + \tan^2 x = \sec^2 x \\
1 + \cot^2 x = \csc^2 x
\]

Odd/Even Identities:

\[
\sin(-x) = -\sin x \quad \tan(-x) = -\tan x \quad \cos(-x) = \cos x \\
\csc(-x) = -\csc x \quad \cot(-x) = -\cot x \quad \sec(-x) = \sec x
\]

Double Angle Formulas

\[
\cos 2x = \cos^2 x - \sin^2 x \\
\quad = 2\cos^2 x - 1 \\
\quad = 1 - 2\sin^2 x \\
\sin 2x = 2\sin x \cos x
\]

Half Angle formulas (BC Only)

\[
\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2} \\
\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}
\]

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<th>45°</th>
<th>60°</th>
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<tr>
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<td>(\sqrt{3})</td>
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General rule of arithmetic sequence: \( a_n = a_1 + d(n-1) \)

Sum of an arithmetic series: \( S_n = n \left( \frac{a_1 + a_n}{2} \right) \)

General rule of a geometric sequence: \( a_n = a_1 (r)^{n-1} \)

Sum of a geometric series: \( S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right) \)

Determine if the sequence is arithmetic, geometric or neither. If the sequence is arithmetic, calculate the common difference and find the formula for the nth term. If it is geometric, calculate the common ratio and find the formula for the nth term.

46. 6, 24, 96, 384,...  
47. 1, 3, 7, 13,...  
48. 4, 13, 22, 31,...  
49. \( \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \ldots \)

50. Find the sum of the first 8 terms of the geometric series if \( a_1 = 6 \) and \( r = 3 \).

51. Find the sum of the first 10 terms of the geometric series 3, 9, 27, ...

Find the sum of the first 20 terms of the series.

52. 34 + 25 + 16 + 7 + ...  
53. 1 + 2 + 4 + 8 + ...

54. \[ \sum_{i=1}^{\infty} \left( 5 \left( \frac{2}{3} \right)^{i-1} \right) \]

Use sigma notation to write each sum.

55. \[ \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots \]

56. \( 2 + 5 + 8 + 11 + \ldots + 29 \)

57. \(-1 + 2 + 7 + 14 + 23 + \ldots + 62 \)

58. \(-2 + 2 - 2 + 2 - 2 + \ldots \)
Polar Formulas:

\[ x = r \cos \theta \quad x^2 + y^2 = r^2 \]
\[ y = r \sin \theta \quad \tan \theta = \frac{y}{x} \]

Convert from Polar to rectangular. Leave answers as exact values.

59. \( \left( -3, \frac{3\pi}{4} \right) \)  
60. \( \left( -4, \frac{11\pi}{6} \right) \)

Convert from rectangular to polar. Round answers to 3 decimals.

61. \((5, 12)\)  
62. \((-3, 5)\)

Change the following equations from rectangular to polar form. Solve for \(r\).

63. \(x^2 = 4y\)  
64. \(y = 3\)  
65. \((x^2 + y^2)^2 = ax^2y\)

Change the following equations from polar form to rectangular form.

66. \(r = 7 \csc \theta\)  
67. \(r = 16 \sin \theta\)

Parametric Equations

68. Sketch the curve \(x = t^2 - 2t\) when \(-2 \leq t \leq 4\).

\(y = t + 1\)

69. Eliminate the parameter to find the Cartesian equation of the parametric curve \(x = t^2\) \(y = 6 - 3t\)