Optional Summer Practice

for

Students Registered for Algebra 2
(2016-2017 school-year)

This optional summer packet was designed to help students prepare for Algebra 2. Algebra 2 is a rigorous course; having a strong foundation in Algebra 1 is the first step toward success! Students, please use the worked examples to guide your practice of the Algebra 1 concepts. Solutions have been provided so that students may check their work.

Online resources for extra practice:

www.khanacademy.org/
http://mathforum.org/
http://education.jlab.org/solquiz/
1) Operation with fractions: Perform the indicated operation and simplify

**Example (Multiply)**

<table>
<thead>
<tr>
<th>Given</th>
<th>Practice:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{7}{3} \cdot \frac{5}{8} )</td>
<td>a) ( \frac{41}{12} \cdot \frac{12}{8} \cdot \frac{11}{11} )</td>
</tr>
<tr>
<td>( \frac{22}{3} \cdot \frac{5}{8} )</td>
<td>b) ( \frac{9}{14} \cdot \frac{28}{45} )</td>
</tr>
<tr>
<td>( \frac{11}{3} \cdot \frac{5}{4} )</td>
<td>c) ( \frac{20}{9} \div \frac{3}{1} \cdot \frac{1}{3} )</td>
</tr>
<tr>
<td>( \frac{11}{3} \cdot \frac{5}{4} )</td>
<td>d) ( \frac{8}{3} \div \frac{5}{12} )</td>
</tr>
<tr>
<td>( \frac{55}{12} \cdot \frac{4}{7} )</td>
<td>e) ( \frac{5}{8} + \frac{7}{6} )</td>
</tr>
<tr>
<td>( \frac{20}{1393} \cdot \frac{5}{7} \cdot \frac{3}{12} )</td>
<td>f) ( \frac{10}{6} - \frac{4}{3} \cdot \frac{2}{3} )</td>
</tr>
</tbody>
</table>

**Example (Divide)**

<table>
<thead>
<tr>
<th>Given</th>
<th>Practice:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{6}{4} \div \frac{3}{2} )</td>
<td>a) ( \frac{4}{12} \div \frac{12}{8} \cdot \frac{11}{11} )</td>
</tr>
<tr>
<td>( \frac{25}{4} \div \frac{15}{2} )</td>
<td>b) ( \frac{9}{14} \div \frac{28}{45} )</td>
</tr>
<tr>
<td>( \frac{25}{4} \div \frac{2}{15} )</td>
<td>c) ( \frac{20}{9} \div \frac{3}{1} \cdot \frac{1}{3} )</td>
</tr>
<tr>
<td>( \frac{5}{2} \div \frac{3}{2} )</td>
<td>d) ( \frac{8}{3} \div \frac{5}{12} )</td>
</tr>
<tr>
<td>( \frac{5}{1} \cdot \frac{1}{3} )</td>
<td>e) ( \frac{5}{8} + \frac{7}{6} )</td>
</tr>
<tr>
<td>( \frac{5}{6} \div \frac{3}{12} \cdot \frac{3}{12} )</td>
<td>f) ( \frac{10}{6} - \frac{4}{3} \cdot \frac{2}{3} )</td>
</tr>
</tbody>
</table>

**Example (Add/Subtract)**

<table>
<thead>
<tr>
<th>Given, re-write as improper fractions</th>
<th>Practice:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{3}{2} + \frac{5}{3} )</td>
<td>a) ( \frac{4}{12} + \frac{12}{8} - \frac{11}{11} )</td>
</tr>
<tr>
<td>( \frac{7}{2} + \frac{17}{3} )</td>
<td>b) ( \frac{9}{14} - \frac{28}{45} )</td>
</tr>
<tr>
<td>( \frac{7}{2} \left( \frac{3}{3} + \frac{17}{2} \right) )</td>
<td>c) ( \frac{20}{9} \div \frac{3}{1} \cdot \frac{1}{3} )</td>
</tr>
<tr>
<td>( \frac{21}{6} + \frac{34}{6} )</td>
<td>d) ( \frac{8}{3} \div \frac{5}{12} )</td>
</tr>
<tr>
<td>( \frac{21}{6} + \frac{34}{6} )</td>
<td>e) ( \frac{5}{8} + \frac{7}{6} )</td>
</tr>
<tr>
<td>( \frac{55}{6} \div \frac{9}{1} \cdot \frac{1}{6} )</td>
<td>f) ( \frac{10}{6} - \frac{4}{3} \cdot \frac{2}{3} )</td>
</tr>
</tbody>
</table>

- re-write mixed numbers as improper fractions
- reduce common factor(s) from top & bottom
- multiply tops & multiply bottoms (separately)
- write as improper fraction or mixed number
- keep 1st fraction, multiply by the reciprocal
- multiply each fraction (as needed) by "1"
- to obtain common denominators
- add / subtract the numerators
- write as improper fraction or mixed number
2) Use long division to find the quotient.

\[
\begin{array}{c|cc}
& 238 & 16 \\
\hline
4) & 954 & 16 \overline{)5052} \\
- & -8 & -48 \\
\hline
- & 15 & 25 \\
- & 12 & -16 \\
\hline
- & 34 & 92 \\
- & -32 & -80 \\
\hline
- & 2 & 12 \\
\hline
\end{array}
\]

\[
954 \div 4 = 238 \frac{2}{4} = 238 \frac{1}{2} \\
5052 \div 16 = 315 \frac{12}{16} = 315 \frac{3}{4}
\]

3) VA SOL A1.1: The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Use the given values to evaluate the expression.

Example:

\[
6x - 2y \quad \text{given}\n\]

\[
6(2) - 2(-5) \quad \text{substitute the values given}\n\]

\[
12 + 10 \quad \text{follow order of operations}\n\]

\[
22 \quad \text{"PEMDAS"}\n\]

Practice:

a) Evaluate the expression if \( x = 2 \) and \( y = -5 \)

\[ -2x^2 + y^3 - z \quad \text{given} \]

\[ -2(-3)^2 + (-1)^3 - (-2) \quad \text{substitute the values given} \]

\[ -18 - 1 + 2 \quad \text{follow order of operations} \]

\[ -17 \quad \text{"PEMDAS"} \]

b) Evaluate the expression if \( x = 3 \), \( y = -1 \), and \( z = -2 \)

\[ 2x^2 \quad \frac{y}{y + 3z} \]

4) VA SOL A1.2a: The student will perform operations on polynomials, including applying the laws of exponents to perform operations on expressions. Use properties of exponents to simplify the expression.

Examples:

a) \( x^5 \cdot x^3 = x^{5+3} = x^8 \) to multiply like bases, add the powers

b) \( \frac{x^{10}}{x^4} = x^{10-4} = x^6 \) to divide like bases, subtract the powers

c) \( 2x^3 = \frac{2}{x^3} \) simplify negative powers by reciprocating the base

d) \( (x^7)^2 = x^{14} \) for a power to a power, multiply the powers

e) \( (-3x^{-3})^2 = 9x^{-6} = \frac{9}{x^6} \) for products and quotients to a power,

f) \[ \frac{2x^2 y^3}{3y^5} = \frac{8x^6}{27y^{15}} \quad \text{raise all parts of the base} \]

Practice:

a) \( x^4 \cdot x^2 = \)

b) \( \frac{x^{12}}{x^3} = \)

c) \( 3x^{-1} = \)

d) \( (x^4)^5 = \)

e) \( (-4x^{-6})^2 = \)

f) \( \left( \frac{3x^4}{5y^3} \right)^2 = \)
5) VA SOL A1.2b: The student will perform operations on polynomials, including adding, subtracting, multiplying, and dividing polynomials. Perform the indicated operation and simplify the expression.

<table>
<thead>
<tr>
<th>Examples:</th>
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</tr>
</thead>
<tbody>
<tr>
<td>a) For addition &amp; subtraction, combine like terms &lt;br&gt; ((3x^3 - 4x^2 + 7x - 8) - (6x^3 + 9x^2 - 2x + 5)) &lt;br&gt; = (3x^3 - 4x^2 + 7x - 8 - 6x^3 - 9x^2 + 2x - 5) <strong>distributive</strong> &lt;br&gt; = (3x^3 - 6x^3 - 4x^2 - 9x^2 + 7x + 2x - 8 - 5) <strong>commutative</strong> &lt;br&gt; = (-3x^3 - 13x^2 + 9x - 13) <strong>substitution</strong></td>
<td>a) ((-3x^3 - 2x^2 + 5x + 4) + (-2x^3 + 7x - 6))</td>
</tr>
<tr>
<td>b) ((2x - 1)(x + 6)) &lt;br&gt; = ((2x \cdot x) + (2x \cdot 6) + (-1 \cdot x) + (-1 \cdot 6)) <strong>distributive &quot;FOIL&quot;</strong> &lt;br&gt; = (2x^2 + 12x - 1x - 6) <strong>substitution</strong> &lt;br&gt; = (2x^2 + 11x - 6) <strong>substitution</strong></td>
<td>b) ((3x^3 - 12x^2 - 5x + 1) - (x^3 - x^2 + 5x + 8))</td>
</tr>
<tr>
<td>c) ((3x + 2)(x - 4))</td>
<td>c) ((3x + 2)(x - 4))</td>
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6) VA SOL A1.2c: The student will perform operations on polynomials, including factoring completely first- and second-degree binomials and trinomials in one or two variables. Factor completely.

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<tr>
<td>a) Greatest Common Factor (GCF) &lt;br&gt; (12x^2 - 8x) &lt;br&gt; = (4x\left( \frac{12x^2}{4x} - \frac{8x}{4x} \right)) <strong>leave the GCF outside</strong> &lt;br&gt; = (4x(3x - 2)) <strong>divide terms by the GCF</strong></td>
<td>a) (10x + 35x^2)</td>
</tr>
<tr>
<td>b) Difference of Squares &lt;br&gt; (a^2 - b^2 = (a + b)(a - b)) &lt;br&gt; (y^2 - 81 = (y + 9)(y - 9)) &lt;br&gt; (25x^2 - 36 = (5x - 6)(5x + 6))</td>
<td>b) (9x^2 - 16)</td>
</tr>
<tr>
<td>c) Quadratic Trinomial &lt;br&gt; (2x^2 + 3x - 20) &lt;br&gt; (a = 2, b = 3, c = -20) &lt;br&gt; <em>What multiplies to (a \cdot c) and adds to (b)?</em> &lt;br&gt; <em>What multiplies to (-40) and adds to (3)?</em> &lt;br&gt; (2x^2 + 8x - 5x - 20) <strong>replace the linear term</strong> &lt;br&gt; ((2x^2 + 8x) - (5x - 20)) <strong>group the terms</strong> &lt;br&gt; (2x(x + 4) - 5(x + 4)) <strong>take out the gcf per group</strong> &lt;br&gt; ((x + 4)(2x - 5)) <strong>factor out the binomial gcf</strong></td>
<td>c) (x^2 - 10x - 24)</td>
</tr>
<tr>
<td>d) Quadratic Trinomial (Perfect Square Trinomial) &lt;br&gt; (x^2 - 20x + 100) &lt;br&gt; <em>What multiplies to (100) and adds to (-20)?</em> &lt;br&gt; ((x-10)(x-10)) &lt;br&gt; ((x-10)^2)</td>
<td>d) (3x^2 - 20x + 12)</td>
</tr>
<tr>
<td>e) (x^2 + 16x + 64)</td>
<td>e) (x^2 + 16x + 64)</td>
</tr>
<tr>
<td>f) (2x^2 + 8x - 42)</td>
<td>f) (2x^2 + 8x - 42)</td>
</tr>
</tbody>
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7) VA SOL A1.3: The student will express the square roots and cube roots of whole numbers and the square root of a monomial algebraic expression in simplest radical form. Simplify the radical expression

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<tr>
<td>a) ( \sqrt{12} ) given&lt;br&gt;( = \sqrt{2 \cdot 6} ) rewrite radicand as a product of factors&lt;br&gt;( = \sqrt{2 \cdot 2 \cdot 3} ) continue to write as product of prime factors&lt;br&gt;( = \sqrt{2^2 \cdot 3} ) look for &quot;pairs&quot; since ( \sqrt{a^2} = a )&lt;br&gt;( = 2\sqrt{3} )</td>
<td>a) ( \sqrt{45} = )</td>
</tr>
<tr>
<td>b) ( \sqrt{90} = \sqrt{3 \cdot 30} = \sqrt{3 \cdot 3 \cdot 10} = \sqrt{3 \cdot 2 \cdot 5 \cdot 3} = \sqrt{3^2 \cdot 2 \cdot 5} = 3\sqrt{10} )</td>
<td>b) ( \sqrt{72} = )</td>
</tr>
<tr>
<td>c) ( 5\sqrt{18} = 5\sqrt{2 \cdot 9} = 5\sqrt{2 \cdot 3 \cdot 3} = 5\sqrt{2^3} = 5 \cdot 3\sqrt{2} = 15\sqrt{2} )</td>
<td>c) ( -2\sqrt{96} )</td>
</tr>
</tbody>
</table>

8) VA SOL A1.4d: The student will solve multistep linear and quadratic equations in two variables, including solving multistep linear equations algebraically and graphically. Solve for \( x \) in the given equation.

<table>
<thead>
<tr>
<th>Examples:</th>
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<tbody>
<tr>
<td>a) Linear ( 5x + 3 - 2(4 - x) = \frac{1}{3}(3x + 12) )&lt;br&gt;( 5x + 3 - 8 + 2x = x + 4 ) distribute&lt;br&gt;( 5x + 2x + 3 - 8 = x + 4 ) commutative&lt;br&gt;( 7x - 5 = x + 4 ) substitution&lt;br&gt;( -x - x ) subtraction&lt;br&gt;( 6x - 5 = 4 ) addition&lt;br&gt;( + 5 + 5 )&lt;br&gt;( 6x = 9 )&lt;br&gt;( x = \frac{9}{6} ) division&lt;br&gt;( x = \frac{3}{2} ) substitution&lt;br&gt;Check : ( 5\left(\frac{3}{2}\right) + 3 - 2\left(4 - \left(\frac{3}{2}\right)\right) = \frac{1}{3}\left(3\left(\frac{3}{2}\right) + 12\right) \to 5.5 = 5.5 )</td>
<td>a) ( -10 + 2x + 3(5 - x) = \frac{1}{2}(4x - 8) )&lt;br&gt;( x^2 - 6x - 72 = 0 )&lt;br&gt;( \frac{3x + 5}{4} = \frac{x}{3} ) (hint: cross-multiply)</td>
</tr>
<tr>
<td>b) Quadratic ( x^2 + x - 12 = 0 ) given (must be set = 0)&lt;br&gt;( (x + 4)(x - 3) = 0 ) factor&lt;br&gt;( x + 4 = 0 ) ( x - 3 = 0 ) set each factor = 0 and solve for ( x )&lt;br&gt;( -4 - 4 ) + 3 + 3&lt;br&gt;( x = -4 ) or ( x = 3 )&lt;br&gt;Note: the real solutions are the ( x )-intercepts of the graph&lt;br&gt;Check : ( (-4)^2 + (-4) - 12 = 0 ) also ( (3)^2 + (3) - 12 = 0 )</td>
<td></td>
</tr>
</tbody>
</table>
9) VA SOL A1.4c: The student will solve multistep linear and quadratic equations in two variables, including solving quadratic equations algebraically. Use the Quadratic Formula to solve the equation.

Example: \(3x^2 - 6x = -2\)

\[3x^2 - 6x + 2 = 0\]

\[a = 3, \ b = -6, \ c = 2\]

**Quadratic Formula**: \[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}\]

\[x = \frac{6 \pm \sqrt{36 - 24}}{6}\]

\[x = \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6}\]

\[x = \frac{3 \pm \sqrt{3}}{3}\]

\[\frac{3 + \sqrt{3}}{3} \approx 1.58\]

\[\frac{3 - \sqrt{3}}{3} \approx -0.42\]

or \[x = \frac{6}{6} \pm \frac{2\sqrt{3}}{6} = 1 \pm \frac{\sqrt{3}}{3}\]

\[\frac{1 + \sqrt{3}}{3} \approx 1.58\]

\[\frac{1 - \sqrt{3}}{3} \approx -0.42\]

Practice:

a) \(x^2 + 4x - 6 = 0\)

b) \(5x^2 + 2x - 2 = 0\)

10) VA SOL A1.4e: The student will solve multistep linear and quadratic equations in two variables, including solving systems of two linear equations in two variables algebraically. Solve each system of equations.

Example:

\[\begin{align*}
4x + 2y &= 20 \\
y &= x - 2
\end{align*}\]

**Substitution method**

\[\begin{align*}
4x + 2(x - 2) &= 20 \\
4x + 2x - 4 &= 20 \\
6x &= 24 \\
x &= 4 \\
y &= x - 2 \\
y &= 4 - 2 \\
y &= 2 \\
(4, 2)
\end{align*}\]

**Elimination method**

\[\begin{align*}
4x + 2y &= 20 \\
y &= x - 2 \\
-x + y &= -2 \\
4x + 2y &= 20 \\
(-x + y &= -2) \times 2 \\
y &= 2 \\
4x + 2y &= 20 \\
x - 2y &= 4
\end{align*}\]

**Check**

\[\begin{align*}
4x + 2y &= 20 \\
y &= x - 2 \\
4(4) + 2(2) &= 20 \\
(2) &= (4) - 2
\end{align*}\]

Practice:

a) \[\begin{align*}
2x + y &= 8 \\
x &= y + 10
\end{align*}\]

b) \[\begin{align*}
3x - 4y &= -5 \\
x - 3y &= 5
\end{align*}\]
11) VA SOL A1.4b: The student will solve multistep linear and quadratic equations in two variables, including justifying steps used in simplifying expressions and solving equations, using field properties and axioms of equality that are valid for the set of real numbers and its subsets. Use the list of properties to name the property illustrated in each statement.

**Examples:**

a. **Addition/Subtraction Property of Equality:**
   
   If \( x = 2 \), then \( x + 3 = 2 + 3 \)

b. **Additive Identity Property:** \( 5 + 0 = 5 \)

c. **Additive Inverse Property:** \( 4 + (-4) = 0 \)

d. **Associative Property of Addition:** \( 2 + 3 + 4 = 2 + (3 + 4) \)

e. **Associative Property of Multiplication:** \( 2 \cdot 3 \cdot 4 = 2 \cdot (3 \cdot 4) \)

f. **Commutative Property of Addition:** \( 8 + 4 = 4 + 8 \)

g. **Commutative Property of Multiplication:** \( -7 \cdot 4 = 4 \cdot -7 \)

h. **Distributive Property:** \( 4(2x - 3) = 8x - 12 \)

i. **Multiplication/Division Property of Equality:**
   
   If \( x = 9 \), then \( 2x = 2 \cdot 9 \)

j. **Multiplicative Identity Property:** \( 24 \cdot 1 = 24 \)

k. **Multiplicative Inverse Property:** \( \frac{4 \cdot 3}{3 \cdot 4} = 1 \)

l. **Reflexive Property of Equality:** \( 6x = 6x \)

m. **Substitution Property:**
   
   If \( x = 3 \) and \( x + 7 = y \), then \( 3 + 7 = y \)

n. **Symmetric Property of Equality:**
   
   If \( 3 + x = y \), then \( y = 3 + x \)

o. **Transitive Property of Equality**
   
   If \( P = 2L + 2W \) and \( 2L + 2W = 60 \), then \( P = 60 \)

p. **Zero Product Property**
   
   If \( x \cdot y = 0 \), then \( x = 0 \) or \( y = 0 \) or both

**Practice:**

1. \( 3(6y) = (3 \cdot 6)y \)

2. \( 23 + 25 + 7 = 23 + 7 + 25 \)

3. \( 1 \cdot 3y = 3y \)

4. \( \frac{x}{3} + 0 = \frac{x}{3} \)

5. \( (5y + 3x) + 10x = 5y + (3x + 10x) \)

6. \( \frac{x \cdot y}{y \cdot x} = 1 \)

7. \( v(3t) = (3t)v \)

8. \(-26x + 26x = 0\)

9. \( 4(3x - 7) = 12x - 28 \)
12) VA SOL A1.5ab: The student will solve multistep linear inequalities in two variables, including solving multistep linear inequalities algebraically and graphically; justifying steps used in solving inequalities, using axioms of inequality and properties of order that are valid for the set of real numbers and its subsets.

Solve the inequality. Show the solution on a number line.

**Example:**

\[5(x + 6) > 7x + 16\]

\[5x + 30 > 7x + 16\]  \textit{distributive}  \\
\[-7x \quad -7x\]  \textit{subtraction}  \\
\[-2x + 30 > 16\]

\[-30 - 30\]  \textit{subtraction}  \\
\[-2x > -14\]

\[
\frac{-2x}{-2} \quad \frac{-14}{-2}\]  \textit{division}

\textit{when multiplying by a negative, switch the inequality}

\[x < 7\] also \"x is less than 7\", also \[\{x \mid x < 7\}\], also \((\neg \infty, 7)\)

**Practice:**

\[6(x - 4) \geq 21 + x\]

13) Algebra I Standard A.6: The student will graph linear equations and linear inequalities in two variables, including a) determining the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined; and b) writing the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line. Re-write the linear equation in slope-intercept form, \(= mx + b\), then graph the line.

**Examples:**

\[2x - 3y = 18\]

\[-2x\]

\[-3y = -2x + 18\]

\[-3y = -2x + 18\]

\[-3\]

\[
y = \frac{-2x}{-3} + \frac{18}{-3}\]

\[
y = \frac{2}{3} - x - 6\]

\[
y = mx + b\]

\[y-intercept = -6, \ slope = \frac{2}{3}\]

**Practice:**

\[2x + 4y = -8\]

**Note:** Given points \((x_1, y_1)\) and \((x_2, y_2)\)

\[slope = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}\]

\textit{Horizontal lines}: slope = 0; equation is \(y = c\)

\textit{Vertical lines}: slope = undefined; equation is \(x = c\)
Review

#1-10 Perform the operation and simplify. #11-14 Factor completely.

1) \( \frac{8}{15} \cdot \frac{4}{6} \)  
11) \( 6x^2 - 20x \)

2) \( \frac{4}{9} - \frac{1}{6} \)  
12) \( x^2 - 144 \)

3) \( x^6 \cdot x^3 \)  
13) \( x^2 + 14x + 40 \)

4) \( \frac{x^{12}}{x^3} \)  
14) \( 4x^2 + 4x - 15 \)

5) \( (2x^5)^3 \)

#15-18 Solve the equation, inequality, or system.

6) \( (3x^4 + 5x^2 - 4x - 3) + (x^3 - x^2 - 5x + 8) \)  
15) \(-6(x - 3) = 5(x + 6) - 1\)

7) \( (x - 6)(x - 6) \)  
16) \( 2x + 3 < 5(x + 3) \)

8) \( (x - 5)(x + 5) \)  
17) \( x^2 + 4x - 3 = 0 \)

9) \( 4231 \div 15 = \)  
18) \( \begin{cases} 
4x + y = 1 \\
x + 4y = -11 
\end{cases} \)

10) \( \sqrt{20} = \)